FORMS IN MUSIC

An Exploratory Mathematical Study of Indian Musical Patterns

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Abstract

Indian music is basically melodic. The melodic schemes, called *ragas*, are about 400 in number in current practice. There has been a continuous tradition of attempts to classify the *ragas* on the basis of scales they employ and/or their melodic similarities. Not many of these systems have been methodologically strict or sufficiently objective.

We here attempt to introduce a method for the study of melody (*raga*) based on the concepts of cybernetics, information theory and related branches. The *raga* is conceived as a statistically definable melodic pattern. This pattern is analysed in terms of entropy and rank correlation.

From this it is proposed to explore methods of defining forms in melody or *raga* patterns.

The present paper will deal with the static aspect of the probability of occurance of notes.

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1. Introduction

1.1. The problem

The various forms of melody in Indian music—currently called *ragas*—have undergone a number of changes during the course of the long history of this land. We have the various styles of *vedic* recitation, the numerous modal types called *jatis* and the later melodic forms which have come to be known as *ragas*. The *ragas* themselves have grown greatly in number and at present we may estimate the total types to be about 400 in number, including North and South Indian styles. Along with the growth in number, there have been periodic attempts to classify them, either by the scales used by them or by the similarity of melodic patterns. Except *Venkatamakhin* of the 17th c., we do not find any one developing a methodologically consistent theory which could be sufficiently objective, comprehensive and fairly immune from the vagaries of historical changes. His method is based on the types of musical scales, (72 *melakartas*) but gives no procedure to classify *ragas* by their tonal patterns. The method developed by others purporting to use melodic patterns have either been very unsystematic or applicable to some restricted period of musical history.

We present here our attemps to use the methods of cybernetics, communication theory and other related methods for the study of melodic patterns in Indian music. The purpose would to be develope methods which would reveal the similarities and differences between *raga* patterns and to construct a tool which would lend itself to versatile use in the study of melodic music. Necessarily, our efforts are exploratory at this stage. We will examine various techniques which can individually or complementarily describe the major musical patterns; no exhaustive study is attempted.

1.2. The concept of raga

The concept of *raga* is the main foundation of the current Indian music. A *raga* may be thought of as a tonal scheme or theme, more or less strictly governed by certain traditionally accepted constraints. An Indian musician trains himself in the intricacies of these schemes and, as an artist, his work is to show to the best of his imaginative ability the beauties of these tonal schemes, conforming himself to the rules of the traditional grammar. Indian music is very much imbued with the 'virtuoso' approach. The most important rules which determine the formation of a *raga* are :

1. A *raga* should have a definite scale.
2. A *raga* should never omit *Sa*. It must have at least one of the following : *Pa*, *Ma*, ma.
3. A *raga* does not have less than four notes.
4. A *raga* (specially North Indian) has a sonant called the *vadi* or *amsa* in a given tetrachord. Its counter part in the other tetrachord is called the *samvadi*. They are expected to be prominent either by repetition or extension. They are said to have a multiplicity (*bahulatva*). Other notes in the *raga* are used in varying degrees of scarcity (*alpatva*).
5. A *raga* has a definite ascent and descent of tones. It is also characterised by certain tonal groups or movements (*varna*, *chalan*). For it is not sufficient to know only the sonants and the probabilities of occurance of notes. One must be very familiar with the nature of the tonal phrases and idioms—i.e., the tonal transitions.

The first two rules are followed more strictly than the others. The third rule finds a few exceptions. The last two are the ones that concern us here. For they impose constraints on tonal occurance and movement. Though these constraints are accepted by all musicians, they are not inviolable; often they are observed more in the breach. Often they even vary from school to school. But for these restrictions, a musician is free to improvise and therein lies his genius. This is akin to the genius of the poet who creates his poem accepting the traditional constraints of his languages.

It is obvious from the foregone description that a *raga* pattern is not completely determined, but is largely statistical in nature. Our purpose here is to develop a typological analysis of these patterns.

The present paper will deal with the *raga* patterns in terms of tonal *probability*. This is a quantitative measure of rule 4 given above.

2. Data and analysis

2.1. Material

The data available are vast and it is not feasible, at this stage, to give an exhaustive analysis. We will confine ourselves to typical examples mainly from North Indian music. One *raga* of South Indian music is also studied for the sake of comparison.

The material available is of two types. First, is the free improvisation (*alap*). This does not readily lend itself to quantitative analysis for two reasons. (1) in the notational form it does not have a regular time measure and (2) it is not available sufficiently extensively in such a form. The second type comprises songs composed in various *ragas*. These are obtainable in printed score quite extensively. They are set to definite time measure and hence amenable to quantitative analysis. Further, in some rare *ragas* only such songs are available and even highly trained musicians rely on these for their knowledge of the *raga*. In view of these reasons we have taken songs in notation as our material for analysis.

Now, reduction of music to notation quantizes the acoustic material to a very high degree. Hence, the finer details of actual music can not be represented by any score. Our analysis has to be studied with this reservation.

2.2. *Units of enumeration*

The units of enumeration are determined by two definitive characteristics of notes : (1) their pitch (position in the musical scale) and (2) their duration.

Indian music recognises, *for purposes of notation only*, twelve notes. (South Indian music has sixteen names for these). We wish to emphasise here that these twelve notes are to be treated as *pitch regions* or *pitch ranges* : there is no question of temperament and each note includes the finer variations like double flats, double sharps and enharmonic variants. The twelve notes, their Western equivalents (on C scale) and numbers on the twelve-tone scale are given below. The numbers will be used in all further descriptions :

Indian	*Sa*	*ri*	*Ri*	*ga*	*Ga*	*Ma*	*ma*	*Pa*	*dha*	*Dha*	*ni*	*Ni*
Western	C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B
Numbers :	1	2	3	4	5	6	7	8	9	10	11	12

Duration is measured in *matra* (crochet). In South Indian music often *akshara* (quaver) is used.

While introducing the concept of the *raga*, we referred to the multiplicity of a note in the *raga* as a definitive character. It is obvious that the multiplicity of an element (note) determines the probability of that element in the set (*raga*). Therefore, the quantitative definition of the multiplicity of a note in *raga* is its probability of occurence.

The probability of any note in a *raga* is estimated as follows. The number of *matras* for which a note occurs in a song either by repetition or extension is counted, including fractions of *matras*. This is divided by the total duration of the song in *matras*. The ratio gives the probability of occurance of the note in the *raga*.

For the purposes of this exploratory study, we have taken six songs in each of the *ragas* under consideration. The tonal probability for notes in each *raga* are given in Table 1.

TABLE 1

The table gives the probability of occurance of a note in the song selected for enumeration. 6 different songs in each *raga* have been analysed, so that in all there are 36 songs. a, b, c, d, e, f, identify the songs for each *raga* as given in standard scores. For instance, in *raga Asavari*, 0.2589 shows the p_i of note No. 1 in Song *a*.

Note No.	Note Name	Tonal Probability						
		a	b	c	d	e	f	Mean
	Raga : *Asavari*							
1.	*Sa*	0.2589	0.2767	0.2105	0.2678	0.2725	0.1907	0.2462
3.	*Ri*	0.1071	0.1699	0.1382	0.1429	0.1350	0.1779	0.1452
4.	*ga*	0.1339	0.1068	0.1053	0.0982	0.1125	0.1107	0.1068
6.	*Ma*	0.982	0.0874	0.0921	0.0938	0.1275	0.0886	0.0979
8.	*Pa*	0.2232	0.1748	0.3026	0.1786	0.1775	0.2226	0.2132
9.	*dha*	0.1786	0.1553	0.1447	0.2009	0.1338	0.1476	0.1602
11.	*ni*	—	0.0291	0.0066	0.0179	0.0413	0.0615	0.0261
	Raga : *Darbari Kanada*							
1.	*Sa*	0.3412	0.1969	0.2553	0.3026	0.2212	0.3214	0.2731
2.	*Ri*	0.1647	0.1024	0.0851	0.1861	0.1062	0.0918	0.1227
3.	*ga*	0.0765	0.1260	0.0709	0.1029	0.0619	0.0561	0.0824
6.	*Ma*	0.1294	0.1732	0.1064	0.0832	0.1770	0.1378	0.1345
8.	*Pa*	0.1353	0.2165	0.2127	0.0983	0.2212	0.1633	0.1746
9.	*dha*	0.0118	0.0551	0.0284	0.0757	0.0619	0.0510	0.0473
11.	*ni*	0.1412	0.1299	0.2411	0.1513	0.1504	0.1786	0.1654
	Raga : *Bhoop*							
1.	*Sa*	0.2890	0.1328	0.3023	0.4009	0.3227	0.4127	0.3101
3.	*Ri*	0.1503	0.1328	0.2209	0.1802	0.1736	0.1270	0.1641
5.	*Ga*	0.2370	0.3750	0.2442	0.1396	0.2609	0.2063	0.2438
8.	*Pa*	0.1156	0.2813	0.1395	0.1306	0.1384	0.1270	0.1554
10.	*Dha*	0.2081	0.0781	0.0930	0.1486	0.1044	0.1270	0.1265
	Raga : *Deskar*							
1.	*Sa*	0.3480	0.3464	0.3333	0.3810	0.3868	0.4235	0.3692
3.	*Ri*	0.0759	0.1000	0.1111	0.1111	0.0660	0.0941	0.0930
5.	*Ga*	0.0928	0.1222	0.1358	0.0952	0.1980	0.1412	0.1309
8.	*Pa*	0.2743	0.2222	0.2099	0.2063	0.1887	0.1706	0.2120
10.	*Dha*	0.2110	0.2111	0.2099	0.2063	0.2170	0.1706	0.2043

Continued

Raga : *Yaman*

1.	*Sa*	0.1789	0.2190	0.2165	0.2000	0.2500	0.3130	0.2296
3.	*Ri*	0.1368	0.1356	0.1443	0.1400	0.1190	0.2408	0.1527
5.	*Ga*	0.1579	0.1074	0.1546	0.2250	0.1667	0.0401	0.1419
7.	*ma*	0.1263	0.1178	0.0825	0.0100	0.0833	0.0369	0.0761
8.	*Pa*	0.1947	0.2742	0.2371	0.2900	0.2143	0.2536	0.2440
10.	*Dha*	0.1105	0.0730	0.0773	0.0880	0.0595	0.0417	0.0737
12.	*Ni*	0.0947	0.0730	0.0576	0.0550	0.1072	0.0731	0.0819

Raga : *Kalyani*

1.	*Sa*	0.1595	0.1872	0.1696	0.2360	0.2033	0.1308	0.1811
3.	*Ri*	0.0859	0.1608	0.1246	0.1164	0.1035	0.0962	0.1146
5.	*Ga*	0.1595	0.1608	0.1280	0.1340	0.0967	0.1923	0.1452
7.	*ma*	0.1718	0.0969	0.1194	0.0893	0.1141	0.1231	0.1191
8.	*Pa*	0.1656	0.1608	0.1488	0.1786	0.2399	0.1423	0.1727
10.	*Dha*	0.1227	0.1564	0.1782	0.1499	0.1364	0.1923	0.1560
12.	*Ni*	0.1350	0.0771	0.1315	0.0957	0.1047	0.1231	0.1112

2.3. *Analysis*

It is necessary to establish that songs included in the same *raga* do have similar structure. That is, they should show reasonable resemblances of belonging to the same population. To establish this the mean distribution of each note in the songs was calculated : these mean values are given in the last column of Table 1. Applying **Chi**^{2} test, it was found that in all cases the individual values did not vary significantly from the means. This goes to show that (a) all the specimens under one head to belong to the same *raga,* hence (b) the *raga* structure itself is statistically definable and (c) the mean distribution can be taken to be representative of the *raga* structure. The **Chi**^{2} values are given in Table 2.

TABLE 2

Chi^2 values for ***ragas.*** Each song is compared with the mean. Table values for all heptatonic *ragas* (*Darbari, Asavari, Yaman, Kalyani*): 11.070 (at 5%).

Table values for all pentatonic ragas (*Bhoop, Deskar*): 7.815 (at 5%)*

Raga	Song No.					
	a	b	c	d	e	f
Asavari	0.04621	0.01652	0.05944	0.02133	0.03196	0·07032
Darbari	0.07090	0.07779	0.07067	0.11221	0.04892	0·02687
Bhoop	0.06562	0.29846	9.03036	0.08052	0.00798	0.05329
Deskar	0.03422	0.00220	0.00737	0.01381	0.04642	0.02245
Yaman	0.07811	0.03748	0.00365	0.52900	0.02843	0.18942
Kalyani	0.04699	0.03571	0.01437	0.01437	0.04920	0.05002

*If p_1, p_2..................p_n are the probabilities observed in π_n cells (a, b, c, d, etc., in Table 1) and π_{n_1}, π_{n_2}........π_n are the ones calculated according to a given hypothesis (Mean values in Table 1) the agreement between the observed aad calculated ones is tested by considering the probabilitv of **Chi**^{2}. It has been shown that $\frac{\Sigma(O-E)^2}{E}$ is distributed as

Chi^2 with (n—1) d.f., where n is the total number of classes, O the observed probability and E the calculated probability. The table values (as obtained from Standard tables) give the values of ***Chi***^{2} for 5% and 1% levels of probability.

The histograms, herein called ***raga-spectra***, are drawn from the mean distributions. (See Figure at end).

3. Definitions of distinction

It was seen above that the tonal probability of notes in a *raga* define it. It is now necessary to study the interrelations of notes within a *raga*, so that these relations give us the tool wherewith further to define and distinguish *ragas*. The problem of distinction becomes important when we consider the fact that the same scale of notes can be so arranged to give different *ragas*, as will be evident from the example below. Further, at least theoretically, the same probabilities distributed differently among the notes of the same scale can also give different *ragas*. The following methods of distinction among *ragas* are introduced to describe the interrelations of notes in a *raga*.

3.1. Entropy

This is a measure of the predictability of the elements in a set—i.e., of the notes in a *raga*. Obviously, the greater the entropy the greater the number of patterns obtainable in a *raga*. This quantity would, therefore, give us a measure of the constraint in a *raga* and thence differentiate one *raga* from another. Table 3 gives the maximum entropy (E), the actual entropy (H), and relative entropy (R) for each *raga* as calculated from the mean distribution in Table 1. (For convenience logs to base 10 have been used.)

TABLE 3

Raga	E	H	R
Asavari	0.8451	0.7903	0.9351
Darbari	0.8451	0.6752	0.9659
Bhoop	0.6990	0.6752	0.9659
Deskar	0.6990	0.6551	0.9372
Yaman	0.8451	0.7988	0.9452
Kalyani	0.8451	0.8377	0.9913

Entropy here is an *empirical measure* and is defined as

$E = -\Sigma p_{i(.)} \log p_i$ where p_i is the probability of an element. It would indicate the extent of scatter of elements in a *raga* and the capacity to generate a number of melodic patterns. The full musical implication of entropy measurements of Indian musical thema will have to be worked out in due course.

The following inferences are in order.

Bhoop and Deskar :

Both these are pentatonic *ragas* with notes 1, 3, 5, 8, 10. Therefore, the maximum entropy (E) for both is the same. They, as scales, are equally capable of providing melodic patterns. But their actual entropies (H) are different, being greater for *Bhoop.* Hence it is less redundant than *Deskar* and has the capacity to have greater melodic patterns.

Darbari and Asavari :

These two are heptatonic *ragas* with notes 1, 3, 4, 6, 8, 9, 11. Their E s are equal. H and hence R for *Darbari* is higher. It is obvious that *Asavari* is more restricted in its tonal distribution. The differences, however, are not marked.

Yaman and Kalyani :

The former is a North Indian *raga* and the latter a South Indian one. Both are heptatonic with the notes 1, 3, 5, 7, 8, 10, 12. They have the same E. The H for *Kalyani* is markedly higher than for *Yaman.* Notice that it has a relative entropy of 0.9913 ! This shows that the seven notes are almost equally distributed, i.e., they have almost equal tonal probabilities. Not so *Yaman.* It is more restricted in its tonal distribution. This is evident from the *raga* spectra also.

We even venture to guess that this may be one of the major differences between the South and North Indian styles. The former is capable of greater patterns in the same *raga.* It further implies that North Indian musicians recognise finer tonal distributions within a set (*raga*) in a definitive manner. They create more *ragas* from the same tonal group but the South Indian creates more patterns in the same *raga.* Elsewhere we have distinguished these two styles as ***autocentric*** (South Indian) and ***telecentric*** (North Indian).[3]

Entropy is a measure of the average predictability of elements in a set. It does not reveal the internal ordering within the set. To measure this it is necessary to determine the conditional entropy (the influence of one element on the predictability of another) in the system. However, this also would suffer from the same fundamental draw back of such entropy measurements in that, given the same probability distributions of elements, their rearrangement among the elements does not alter the entropy of the system. Therefore, methods to describe the internal structure of a system are necessary. One such method is described below in 3.2.

3.2. *Rank Correlation :*

The notes used in a *raga* are arranged in the ascending order of pitch. Each is given an ordinal number in this rank ordering. This is its pitch rank. Obviously, rank arrangement according to this scale will be the same for *ragas* which use the same kind of notes. The tonal probabilities of notes are also arranged in an increasing order. This ranking will always be different for different *ragas*, even if they use the same kind of notes. Here then are two rankings : one according to pitch (which may be identical for any number of *ragas*, e.g., for *Bhoop* and *Deskar*) and the other according to tonal strengths (which must always be different for different *ragas*.) The relation between these two rankings may be expressed by the coefficient of rank correlation, τ following Kendall[5]. τ values for the *ragas* are given in Table 4.

TABLE 4

τ, the coefficient of rank correlation

Asavari	*Darbari*	*Bhoop*	*Deskar*	*Yaman*	*Kalyani*
—0.43	—0.14	—0.8	0	—0.43	—0.24

It is evident that a high negative correlation indicates emphasis on the lower tetrachord and a high positive correlation an emphasis on the upper tetrached.

One peculiarity may also be noted. All the *ragas* (except *Deskar*) have a negative correlation, which could mean that they all emphasise the lower tetrachord. Perhaps this is an accidental feature caused by the choice of the *ragas* studied here. It may also be due to the great tonal probability of the note No. 1 which is the universal tonic in modern Indian music. This will be discussed below.

4. Summary and discussion

The present day music of India is based on the *raga* concept. *Ragas* are statistically definable tonal schemes. It is here attempted to explore methods of mathematically defining *raga* patterns.

Six songs in notation for each of the six *ragas* chosen were analysed quantitatively. The *raga* structure is defined in terms of the tonal strengths of the component notes, i.e., in terms of the probability of occurance of the notes in the *ragas*. The interrelations of the tonal strengths in the *raga* are expressed in the following ways.

4.1. Entropy

Employing the probabilities of notes in a *raga*, its entropy is calculated. Knowing the number of notes in the *raga*, its maximum entropy is calculable. From these, the relative entropy of *raga* is also determined. It is found that the entropies of *ragas* which use the same notes are distinguishably different. For instance, pairs like *Bhoop-Deskar*, *Darbari-Asavari*, *Kalyani-Yaman* are distinguished by their entropy characteristics. The entropies of *ragas* also give us an insight into their nature; for entropy in the main indicates the potentiality of a *raga* for generating melodic patterns. Perhaps, this may eventually be related to the ethos of the *raga*.

It is necessary to note here the position of note No. 1 in these distributions. It is doubtful whether this note should be accounted for at all in any determination of *raga* forms. First, in all except one *raga*, herein studied, it has the highest probability of occurance. Hence, perhaps, it does not contribute any differentiating character to a *raga*. Secondly, its strength is often so much greater than those of other notes that it vitiates perspective of the other notes in the calculation. This is specially observable in *Darbari*. This unique position of *Sa* is due to its being the universal keynote and tonic in current Indian music. This might even give us a clue to distinguish current Indian music from the ancient modal one.

4.3. Rank correlation

Whereas entropy shows the predictability of the *raga* form, it has one defect. For, if the tonal strengths are distributed differently among the notes of a given scale, one can get a different *raga*. But the entropy relation will not alter. Under such cases rank correlation provides a measures of differentiation. For it is an expression of the internal relations of notes to their probabilities of occurance.

The coefficient may be taken to indicate the emphasis of the lower or upper tetrachord. Here we may note the incongruous effect of *Sa*, as it is almost always the first in ranking in both pitch and probability scales. Further, since the upper *Sa* is counted with the middle *Sa*, its position mixes up the tetrachords. This is the reason, perhaps, why we have not found a *raga* with a + rank correlation.

References

1. The following standard scores have been used for analysis. Roman numerals indicate volume of the book and the arabic numerals indicate the page numbers on which the songs can be found. Year of publication is given in parenthesis. In each *raga*, there are six songs identified by *a, b, c, d, e, f,* in Table 1.

Bhatkande, V. N., *Hindustani Sangeet Paddhati, Kramik pustak* (Hindi)

I, 19, 20. II. 25, 29, 33, 34 (1947) — *Yaman*
I, 39, 40. II. 364, 365, 361, 371 (1947) — *Asavari*
III, 19, 20, 21, 22, 23, 25 (1950) — *Bhoop*
IV, 241, 243, 244, 245, 246, 247 (1949) — *Deskar*
IV, 651, 652, 653, 655, 656, 657 (1949) — *Darbari*

Rangaramanuj a Iyengar : *Kritimani malai* (Tamil)

III. 2, 399 (1948). IV, 291. (1948). V. 419 (1953) — *Kalyani*

S. Srinivas a Iyer : *Swati Tirunal Kritimalai* (Tamil) 279—*Kalyani*

Lalitangi : *Purandara mani mala,* I, 44 — *Kalyani*

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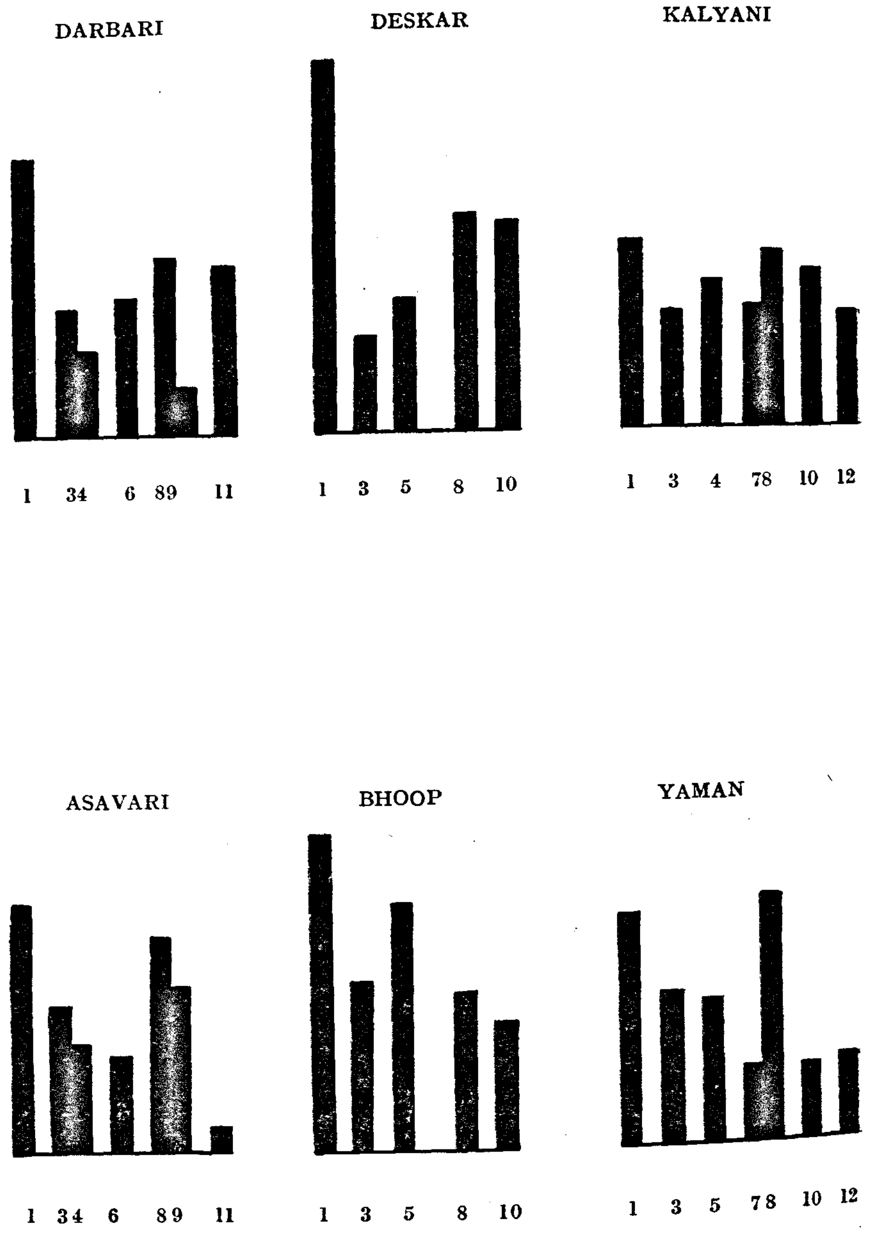

Fig. 1 *Raga* spectra. The numbers on the horizontal axis denote notes in the concerned *raga*. The height of a bar is proportional to the tonal probability of the note in the *raga*. The wider scatter in *Kalyani* (S. Indian) and regions of concentration in the five North Indian *ragas* may be noted.